Name: ..…………………………………………………………………………………

Student Number: ……………………………………………………………………..

Test 4 on WPPH16001.2020-2021
"Electricity and Magnetism"

Content: 5 pages (including this cover page); **4 questions**

Friday April 30 2020; **online**, 14:00-16:00

- Write your full name and student number on **each** page you use
- Read the questions carefully. Read them one more time after having answered them.
- Compose your answers is such a way that it is well indicated which (sub)question they address
- Upload the answer to each question as a **separate pdf file**
- Do not use a red pen (it's used for grading)
- Griffiths' textbook, lecture notes and **your** tutorial notes are allowed. The internet, mobile phones, consulting, requests for consultancy and other teamwork are not allowed and considered as cheating

Exam drafted by (name first examiner) Maxim S. Pchenitchnikov

Exam reviewed by (name second examiner) Steven Hoekstra

For administrative purposes; do NOT fill the table

The weighting of the questions:

	Maximum points	Points scored
Question 1	8	
Question 2	10	
Question 3	15	
Question 4	10	
Total	**43**	

Grade = 1 + 9 x (score/max score).

Grade: ________

Question 1 (8 points)

Old-generation incandescent lamps sometimes burned down right after being switched on but not during their operation. This is curious considering the fact that the switching-on moment is short as compared to long operation time. Let's figure out why it is so.

Consider an incandescent lamp with the power of $P = 230$ W connected to a standard power net with $V = 230$ V voltage (potential difference). The filament of the lamp is made of tungsten, the resistance $R(T)$ of which as a function of temperature T can be approximated as

$$R(T) = R_0 \cdot (0.006T - 0.77),$$

where T is expressed in Kelvin, and R_0 is the filament resistance at $T = 300$ K (i.e. at room temperature). The filament temperature when the lamp operates, is ~ 3000 K so that the filament radiates visible light (and, by the way, a lot of infrared light which makes these lamps very inefficient).

1. Find the filament resistance R when the lamp operates. (2 points)

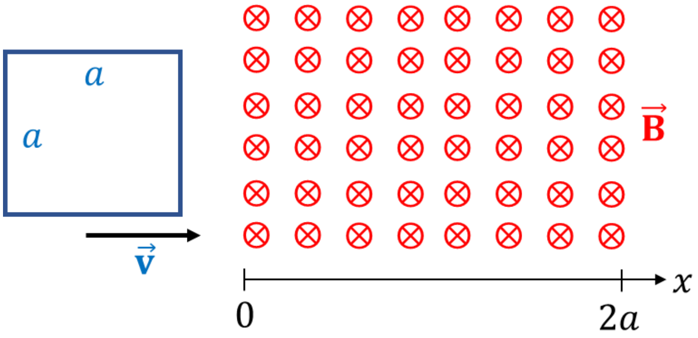
2. Find the filament resistance R_0 when the lamp is off for a long time (i.e. at room temperature). (2 points)

3. Now find the power P dissipated by the filament at the first moment when the lamp is just switched on (i.e. the voltage has been applied but the filament is still at room temperature). (2 points)

4. Now explain why the filament burns down while switched on. (2 points)

Question 2 (10 points)

A square loop of side a and resistance R moves with speed $\vec{\mathbf{v}}$ into a region in which a magnetic field of magnitude $\vec{\mathbf{B}}$ exists perpendicular to the plane of the loop and direction into the screen/paper, as shown in the figure.



Find emf **and** the current direction when the loop:

1. is entering the region with magnetic field (3 points)

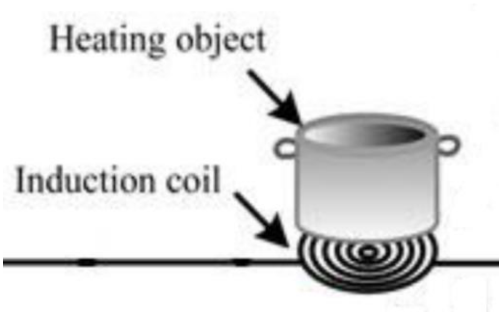
2. is moving through the region with magnetic field (2 points)

3. is exiting the region with magnetic field (1 point)

4. Sketch a plot of the current induced in the loop as it enters, moves through, and exits the region of the magnetic field. For the coordinate of the loop, take the position of its right vertical segment; for the current, consider the clockwise direction positive. Don't forget to mark characteristic lengths along *x*-axis and values of the emf along the *y*-axis. (4 points)

Question 3. (15 points)

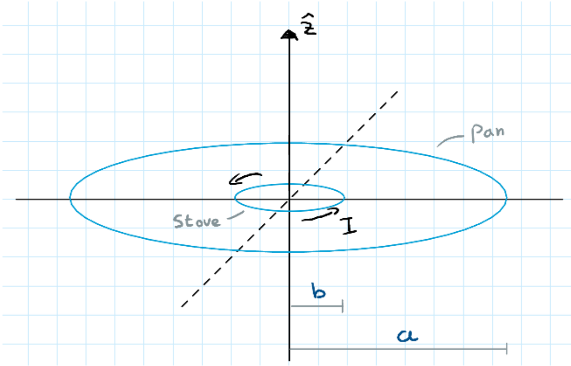
A modern induction stove consists of a coil fed with a rapidly oscillating current right below a glass plate on which a pan of metal is placed just above the coil (but with no contact to it – that's why you need an isolation glass plate).



1. Explain qualitatively how such a stove heats up your food. (2 points)

2. Which material for the pan bottom is better from the viewpoint of heat production: copper or aluminum? Both materials are non-magnetic. (Hint: consult Table 7.1 in Griffiths) (2 points)

The figure to the right shows a simplified schematic of an induction stove where we consider the stove and the pan as single circular loops of wire of radii of a and b respectively. In our model, the pan is much larger than the stove ($a \gg b$) and right on top so that you may neglect any vertical distance between the two (in other words, the two loops lay in the same plane). The pan has internal resistance R.



3. Show that the current $I_P(t)$ that flows through the pan if the stove is fed by a time-dependent (alternative) current $I_S(t) = I_0 \cos(2\pi\nu t)$, where ν is the oscillation frequency of the current, is given as

$$I_P(t) = \frac{\mu_0 \pi^2 b^2 \nu I_0}{Ra} \sin(2\pi\nu t)$$

Hint: (i). use the symmetry property of mutual induction; and (ii). approximate the field to its value at the center (6 points)

4. How long would it take for the stove to dissipate an energy Q into the pan? (4 points)

Hint: you may use the approximation that if the oscillations are fast enough, $\cos^2(2\pi\nu t) \rightarrow 1/2$ and $\sin^2(2\pi\nu t) \rightarrow 1/2$ over an integral in time.

5. Explain why the induction stoves use alternative currents of high frequency $\nu = 24$ kHz to feed the induction coil rather than alternative current directly from the power net at frequency of 50 Hz. (1 point)

Question 4. (10 points)

To directly measure the displacement current, researchers use a time-varying voltage $V(t)$ with angular frequency ω to charge and discharge a circular parallel-plate capacitor with the plate spacing d. The magnetic field inside such a capacitor is given as

$$\vec{\mathbf{B}}(s) = \frac{\mu_0 \epsilon_0 s\ \omega V_0 \cos \omega t}{2d}\ \hat{\boldsymbol{\varphi}}$$

where s is the distance from the center of the capacitor.

1. Show that the displacement current density $\vec{\mathbf{J}}_d = \frac{\epsilon_0 \omega V_0 \cos \omega t}{d} \hat{\mathbf{z}}$ (6 points)

2. Find the electric field $\vec{\mathbf{E}}$ (3 points)

3. Why is a time-varying voltage used but not a constant voltage? (1 point)